

# Performance evaluation of a faster-than-Nyquist system based on turbo equalization and LDPC codes

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# Plan

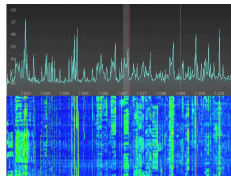
- 1 Scarce radio spectrum resources?
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# Challenges in wireless communications

## High level trade-off in digital communications

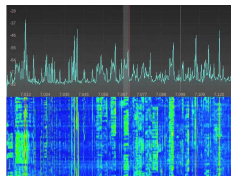
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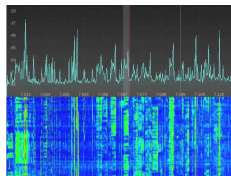
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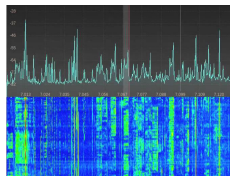


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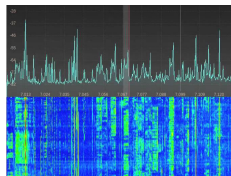
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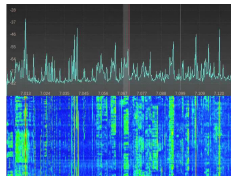
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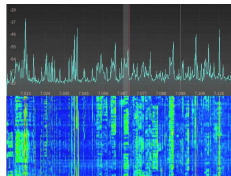
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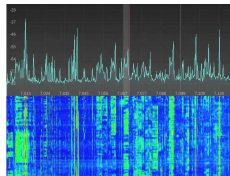
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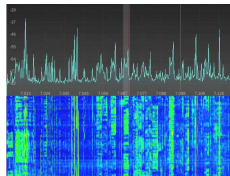
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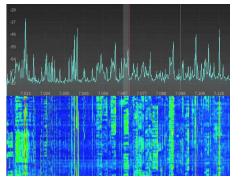
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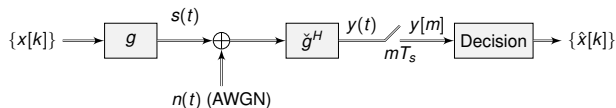
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- Increase of constellation size  $M \uparrow \rightarrow \downarrow d_{sym} \rightarrow \uparrow P_e$
- Decrease of symbol spacing  $T_s \downarrow \rightarrow$  introduction of interference that can be **modeled** and **removed** with a **more complex** receiver.

# Modelling interference



$$s(t) = \sum_{k=-\infty}^{+\infty} x[k] g(t - kT_s), \quad x[k] \in \mathbf{A}, \quad |\mathbf{A}| = M \quad (1)$$

## Noiseless discrete equivalent channel

- $h[l] = (g * \check{g}^H)(lT_s), \quad l \in [0, \dots, L-1]$

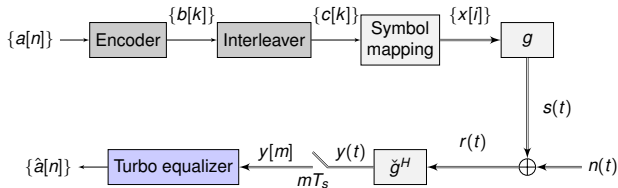
## Interference characterization [3]

- If  $\frac{1}{T_s} \leq B$ , the system can achieve  $h[l] = \delta_{0,l}$  (Nyquist)
- If  $\frac{1}{T_s} > B$ , inevitably  $h[l] \neq \delta_{0,l}$  (Mazo) [2]

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# Faster-than-Nyquist system

Introduction of non-linear processing: **turbo equalization**



**Figure:** Block diagram of the faster-than-Nyquist system [4]



# Turbo equalizer structure

Turbo equalization [5]: iterative exchange [1] between two blocks

- **Equalizer**: two approaches considering  $h[l]$ ,  $l \in \{0, \dots, L-1\}$ 
  - Minimum Mean Square Error (MMSE) (complexity in  $\mathcal{O}(ML)$ )
  - *Maximum a Posteriori* (MAP) (complexity in  $\mathcal{O}(M^L)$ )
- **Decoder**: we use LDPC codes from DVB-S2 standard

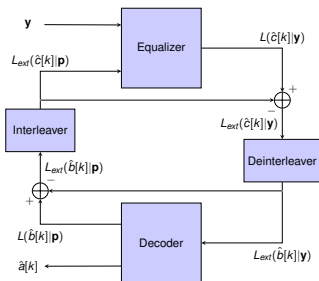


Figure: Turbo equalizer

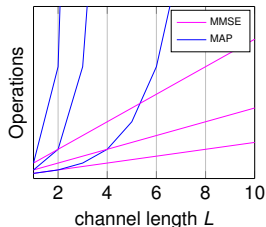


Figure: Equalizer's complexity for  $M = 2, 4, 8$

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# Performance and complexity

## Contribution

- Evaluation of MAP performance with short-length channel response models (truncated channels)
- Comparison between truncated (length  $L'$ ) MAP equalization and (length  $L$ ) MMSE equalization

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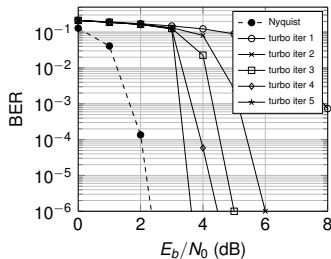
- Evaluation of MAP performance with short-length channel response models (truncated channels)
- Comparison between truncated (length  $L'$ ) MAP equalization and (length  $L$ ) MMSE equalization

Can MAP outperform MMSE while keeping reasonable complexity?

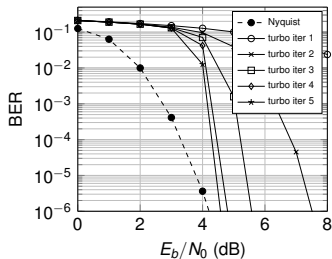
Consider a truncated channel of length  $L' < L$  such that

$$\sum_{m=0}^{L'-1} |h[m]|^2 \leq \beta, \quad \beta \in [0; 1[. \quad (2)$$

## FTN MMSE-LDPC performance



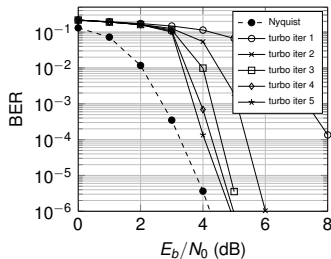
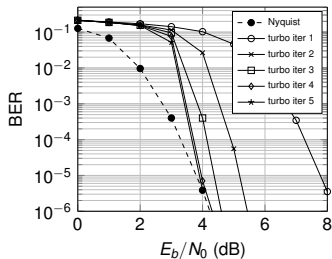
(a) 10 LDPC iterations



(b) 5 LDPC iterations

Figure: FTN MMSE-LDPC performance with  $L = 9$ ,  $\frac{1}{T_s B} = 1.4$

## FTN MAP-LDPC performance

(a) 3-coefficient MAP ( $L' = 3$ )(b) 5-coefficient MAP ( $L' = 5$ )Figure: FTN MAP-LDPC performance with  $\frac{1}{T_s B} = 1.4$

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# Summary

What do we achieve with faster-than-Nyquist?

- 1 **Spectral efficiency gain** in wireless communications systems;
- 2 Truncated MAP can outperform MMSE while keeping reasonable complexity (provided  $2 \leq M \leq 4$ ).



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## What do we achieve with faster-than-Nyquist?

- 1 **Spectral efficiency gain** in wireless communications systems;
- 2 Truncated MAP can outperform MMSE while keeping reasonable complexity (provided  $2 \leq M \leq 4$ ).

## What are the main challenges?

- 1 **Trade-off** between complexity and performance;
- 2 Real-time implementation: study on **synchronization** needed.

# Summary

Thank you for your attention

Do you have any questions?

# References I



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